QUIZ 6 SOLUTIONS: LESSON 7
SEPTEMBER 13, 2017

Write legibly, clearly indicate the question you are answering, and put a box or circle around your final answer. If you do not clearly indicate the question numbers, I will take off points. Write as much work as you need to demonstrate to me that you understand the concepts involved. If you have any questions, raise your hand and I will come over to you.

1. [4 pts] Find the general solution to $\dfrac{dy}{dt} + 3y = 0$

Solution: Write

$$\begin{aligned} \frac{dy}{dt} + 3y &= 0 \\ \Rightarrow \quad \frac{dy}{dt} &= -3y \\ \Rightarrow \quad \frac{1}{y}\, dy &= -3\, dt \\ \Rightarrow \quad \int \frac{1}{y}\, dy &= \int -3\, dt \\ \Rightarrow \quad \ln|y| &= -3t + C \end{aligned}$$

Now, we need to get y by itself. We will assume for this class that $y > 0$ so we drop the absolute values. We also apply e to both sides to get

$$\begin{aligned} e^{\ln(y)} &= e^{-3t+C} \\ \Rightarrow \quad y &= e^{-3t+C} \\ &= e^{-3t}e^{C} \\ &= e^{C}e^{-3t}. \end{aligned}$$

Because we are looking for a general solution, we may replace e^C by C without any issue. Thus, our general solution is

$$\boxed{y = Ce^{-3t}}$$

2. Suppose the number of members of a rat colony grows proportionally to its population. Suppose there are initially 100 rats in the colony and, after 6 months, there are 300 rats.

   (a) [2 pts] Write down a differential equation that represents the change in population of the rats.

   Solution: Because the population is changing proportionally to itself, we have a differential equation of the form

   $$\boxed{P'(t) = kP(t)}$$

   where k is the proportional constant and $P(t)$ is the population of rates at time t, for t measured either in months or years (or even days or minutes if you wish to be difficult; it doesn't matter in this context how you want to measure t).

   (b) [4 pts] How many rats are there after 1 year? Round your answer to the nearest rat.

   By part (a) and comments made in the notes, we know that

   $$P(t) = P(0)e^{kt}.$$

   We are told that $P(0) = 100$, so

   $$P(t) = 100e^{kt}.$$

   Suppose t is time measured in years. Then, we see that $P(.5) = 300$, which means

   $$100e^{k(.5)} = 300.$$

   Thus,

   $$\begin{aligned} 100e^{.5k} &= 300 \\ \Rightarrow \quad e^{.5k} &= 3 \\ \Rightarrow \quad \ln e^{.5k} &= \ln 3 \\ \Rightarrow \quad .5k &= \ln 3 \\ \Rightarrow \quad k &= 2\ln 3. \end{aligned}$$

   The question asks us to find $P(1)$. We write

   $$\begin{aligned} P(1) &= 100e^{(2\ln 3)(1)} \\ &= 100e^{2\ln 3} \\ &= 100e^{\ln 3^2} \\ &= 100(3^2) \\ &= \boxed{900}. \end{aligned}$$

   Note 0.1. If you wanted to measure t in months, that also works perfectly well. You will get the same answer, although your k will be different.